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$GSO(-)$ vertex operators and open superstring field theory in hybrid variables

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Abstract: We consider the $GSO(-)$ sector of the open superstring using the formalism with four-dimensional hybrid variables. This sector is defined by the world sheet hybrid variables $(\theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ with antiperiodic boundary conditions. The corresponding spectrum of states and conditions for physical vertices are described. In particular we construct explicitly the lower level $GSO(-)$ vertex operators corresponding to the tachyon and the massless fermions. Using these new vertices, the tachyon and massless sector contribution to the superstring field theory action of Berkovits is evaluated. In this way we have included the Ramond sector and we end by discussing some features of the action.

Keywords: Tachyon Condensation, String Field Theory, Supersymmetry Breaking.

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Contents

1. Introduction

It has been considered that the more complete framework to study the dynamics of the unstable non-BPS D-branes must be based on open superstring field theory [1]. This theory must include both Gliozzi-Scherk-Olive (GSO) sectors of the superstring that means the standard positive $GSO(+)$ and the negative $GSO(-)$ projected sector. The process of annihilation of these unstable non-BPS D-branes was studied using superstring field theory action involving only the Neveu-Schwarz (NS) sector, without the Ramond (R) sector. However, it is still not clear what is the role of the spacetime supersymmetry during this process. For this reason, it is very interesting to explore some aspects of this process from the point of view of spacetime supersymmetry, as in [2]. Since the spacetime supersymmetry always involves the contribution of both the NS and R sectors of the superstring we need to extend the framework to keep track of the Ramond sector.

There is a formulation of the superstring in terms of hybrid variables which is suitable to explore the issues of space-time supersymmetry. Specifically, we will use the four-dimensional version of this formalism to study both GSO sectors and both R and NS sectors of the superstring that means we consider: $NS+$, $NS-$, $R+$ and $R-$ sectors.

The standard $GSO(+)$ sector has spacetime supersymmetry and it is identified with the periodic sector of the $\theta, \bar{\theta}$ world sheet variables. In this sector all vertex operators are N=1 D=4 superfields due to the fact that the spacetime supersymmetry current has zero modes. A detailed discussion of this sector was given in references [3] of Berkovits.

On the other hand, the $GSO(-)$ sector is not spacetime supersymmetric and it is identified with the antiperiodic sector of the $\theta, \bar{\theta}$ world sheet variables [2]. In this sector the vertex operators are ordinary fields and the spacetime supersymmetry is broken since the supercurrents have half integer modes.

Using these ideas in order to understand both GSO sectors, we proceed as follows.

In section 2 we review the hybrid approach for superstring and explain its relation with the RNS approach. We use the mapping between the two sets of variables to show the vertex operators for tachyon and massless sector in both formalisms. The conditions for physical vertices are also discussed.

In section 3, as a consistency check, we construct the physical $GSO(-)$ vertices that correspond to the tachyon and the massless fermions using the hybrid variables. All these vertices contain spin fields to change to antiperiodic boundary conditions for the $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$ variables.

In section 4 we review the open superstring field theory action which describes non-BPS D-branes using the hybrid variables. This action requires three independent superstring fields represented by Φ, Ψ and $\bar{\Psi}$ which are appropriately tensorized, so as to take into account the two GSO projections.

In section 5, we explicitly compute the classical superstring field theory action including the contribution of the tachyon, the massless fermions, and massless gauge bosons.

The full action, written using explicit 10-dimensional Lorentz invariant fields is:[1]

$$\begin{aligned} S = \mathrm{Tr} \int d^{10}x \Big[& \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + T \Box T + \frac{1}{2} T^2 + [T, A^\mu][T, A_\mu] - T^4 + \\ & + \chi^{a\,+} \gamma^\mu_{ab} \left(\partial_\mu \chi^{b\,+} + [A_\mu, \chi^{b\,+}] \right) + \\ & + \chi^{a\,-} \gamma^\mu_{ab} \left(\partial_\mu \chi^{b\,-} + [A_\mu, \chi^{b\,-}] \right) + 3^{\frac{3}{4}} T (\chi^{a\,+} \chi^-_a) + \cdots \Big] \end{aligned} \tag{1.1}$$

where F_{mn} is the field-strength for the gauge-field A_m, T is the tachyon field, χ^+_a, χ^-_a are the massless fermions coming from the $GSO(+)$ and $GSO(-)$ sector that are 10-dimensional chiral and antichiral respectively.

At the end we comment on some of the features of this action.

[1] Actually, the term with 4 gauge fields in $F^{mn}F_{mn}$ has never been calculated and most likely is not going to give the correct coefficient without going to an effective action. Anyway, we don't use this result anywhere in this work.

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2. Hybrid superstring with both $GSO(+)$ and $GSO(-)$ sectors

In this section we review the relation between RNS and hybrid variables. Using the map between them we show the vertex operators in both formalisms and we discuss the conditions for physical vertex operators.

2.1 RNS variables

First we represent the RNS Matter and ghost variables in the large Hilbert space:

$$\Bigg[\underbrace{x^m,\ \psi^m,\ b,\ c,\ \xi,\ \eta,\ \phi}_{D=4 \text{ space-time matter + ghosts}},\ \underbrace{x^{\pm j},\ \psi^{\pm j}}_{CY}\Bigg],$$

the first set of describes the RNS $D=4$ spacetime matter plus the ghost worldsheet fields and the second set describes the other six dimensional internal worldsheet fields. These last internal fields may be compactified in a Calabi-Yau or not, but in the present case we just complexify them as explained later on in this same paragraph. Specifically we have: x^m : $(m = 0,1,2,3)$, are the $D = 4$ spacetime vector RNS world sheet bosonic coordinates; $x^{\pm j} = \frac{1}{\sqrt{2}}(x^{2j+2} \pm i x^{2j+3})$, $(j = 1,2,3)$. are the internal d=6 RNS world sheet bosonic coordinates; ψ^m, $(m = 0,1,2,3)$, are the $D = 4$ spacetime vector RNS world sheet fermionic coordinates; $\psi^{\pm j} = \frac{1}{\sqrt{2}}(\psi^{2j+2} \pm i\psi^{2j+3})$, $(j = 1,2,3)$ are the internal d=6 RNS world sheet fermionic coordinates. See appendix A about the properties of these RNS variables.

In order to construct space time fermion vertex operator, we need to define the $D = 4$ spacetime RNS spin fields Σ^α and $\bar{\Sigma}^{\dot\alpha}$ by

$$\begin{aligned} \Sigma^\alpha &= \Sigma^{\pm\pm} \quad (even\ +)\,, \\ \bar{\Sigma}^{\dot\alpha} &= \Sigma^{\pm\pm}; \quad (odd\ +)\,, \end{aligned}$$

and the U(1) internal chiral boson H by

$$\begin{aligned} J = \partial H &= i\left(\psi^4\psi^5 + \psi^6\psi^7 + \psi^8\psi^9\right) =: \psi^{+j}\psi^{-j} :, \\ H(z)H(w) &= 3\ln(z-w)\,. \end{aligned}$$

As is well known, the RNS approach of superstring has the problem that the off-shell space-time supersymmetry is not manifest due to the problem of picture [4]. This problem makes it difficult to set up a second quantized approach of superstrings which is gauge invariant and free of contact term divergences at the tree level at the same time. However, as we will discuss in the next subsection, there is a more convenient formulation of the superstring. This alternative formulation is the so called hybrid approach, and preserves off-shell supersymmetry.

2.2 Hybrid variables

In the hybrid approach of the superstring the fundamental variables are the following world sheet fields:

$$\left[X^m,\ \theta^\alpha,\ \bar{\theta}^{\dot\alpha},\ p_\alpha,\ \bar{p}_{\dot\alpha},\ \rho,\ \Gamma^{+j}, \Gamma^{-j},\ X^{+j},\ X^{-j}\right]$$

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- here X^m is the $D=4$ spacetime vector world sheet boson, $m=0,1,2,3$;
- θ^α is the $D=4$ spacetime chiral spinor worldsheet fermion, $\alpha=1,2$;
- $\bar\theta^{\dot\alpha}$ is the space-time anti-chiral spinor world sheet fermion, $\dot\alpha=1,2$;
- ρ is the world sheet chiral boson;
- $\Gamma^{\pm j}$ are internal world sheet fermions, $j=1,2,3$;
- $X^{\pm j}$ are internal world sheet bosons, $j=1,2,3$.

The hybrid variables are defined in such a way that we have two independent set of worldsheet fields. The first set satisfy a $N=2$, $c=-3$ superconformal algebra and describes the $N=1$, $D=4$ super spacetime and the second set satisfy a $N=2$, $c=9$ superconformal algebra and describes the internal coordinates. Even though any CY compactification is allowed for this last set of fields, we are going to consider only the complexification of them in flat space.

In the appendix B, all the properties of the hybrid variables are given. And we should remember that the mapping between RNS and hybrid variables is a unitary transformation as in reference [3]

The non-vanishing OPEs for the hybrid variables are:

$$\begin{aligned}
X^m(z)X^n(w) &= -\eta^{mn}\ln(z-w)\,,\\
p_\alpha(z)\theta^\beta(w) &= \frac{\delta_\alpha^\beta}{z-w}\,,\\
\rho(z)\rho(w) &= -\ln(z-w)\,,\\
\Gamma^{+i}(z)\Gamma^{-j}(w) &= \frac{\delta^{ij}}{(z-w)}\,,\\
X^{+i}(z)X^{-j}(w) &= -\delta^{ij}\ln(z-w)\,.
\end{aligned}\tag{2.3}$$

The hybrid superstring is described by the following quadratic world sheet action

$$S_{hyb}=\int d^2z\left[\frac{1}{2}\partial X^m\bar\partial X_m+p_\alpha\bar\partial\theta^\alpha+\hat p_{\dot\alpha}\bar\partial\hat\theta^{\dot\alpha}+\bar p_\alpha\partial\bar\theta^\alpha+\bar{\hat p}_{\dot\alpha}\partial\bar{\hat\theta}^{\dot\alpha}\right]+S_\rho+S_c\,,$$

where S_ρ, S_c are the actions for the chiral ρ boson and the compact variables respectively. For completeness we included the right-moving world-sheet fields in this action (denoted with a hat). In the following only the left-moving sector is considered.

This world sheet action has a $N=2$ superconformal algebra formed with the generators L, G^+, G^-, J_{gh}. It turns out that is convenient to extend this superconformal algebra to form an $N=4$ superconformal algebra formed by the following generators: J^{--} with conformal weight 0; G^+, $\tilde G^+$, J_{gh} with conformal weight 1 and L, G^-, $\tilde G^-$, J^{++} with conformal weight 2 (here the upper indices $+$ and $-$ indicate the U(1) ghost charge generated by J_{gh}):

$$L=-\frac{1}{2}\partial X^m\partial X_m-p_\alpha\theta^\alpha-\bar p_{\dot\alpha}\bar\theta^{\dot\alpha}-\partial\rho\partial\rho-\frac{1}{2}\partial^2\rho+L_c\,,$$

$$
\begin{aligned}
G^{+} &= \underbrace{e^{\rho}(d)^{2}}_{G^{+}_{0}} + \underbrace{\Gamma^{-j}\partial X^{+j}}_{G^{+}_{-\frac{1}{3}}}\,, \\
\tilde{G}^{+} &= \underbrace{\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}e^{-2\rho}(\bar{d})^{2}}_{G^{+}_{-1}} + \underbrace{e^{-\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\partial X^{-l}}_{G^{+}_{-\frac{2}{3}}}\,, \\
G^{-} &= \underbrace{e^{-\rho}(\bar{d})^{2}}_{G^{-}_{0}} + \underbrace{\Gamma^{+j}\partial X^{-j}}_{G^{-}_{\frac{1}{3}}}\,, \\
\tilde{G}^{-} &= \underbrace{\epsilon^{jkl}\Gamma^{+j}\Gamma^{+k}\Gamma^{+l}e^{+2\rho}(d)^{2}}_{G^{-}_{1}} + \underbrace{e^{\rho}\epsilon^{jkl}\Gamma^{+j}\Gamma^{+k}\partial X^{+l}}_{G^{-}_{\frac{2}{3}}}\,, \\
J_{gh} &= \partial\rho + \Gamma^{-j}\Gamma^{+j}\,, \\
J^{++} &= e^{-\rho-\Gamma^{-j}\Gamma^{+j}}\,, \\
J^{--} &= e^{\rho+\Gamma^{-j}\Gamma^{+j}}\,,
\end{aligned}
\tag{2.4}
$$

where L_c is the stress tensor for the internal variables $\Gamma^{\pm j}$, $X^{\pm j}$ and

$$
d_{\alpha}(z) = p_{\alpha}(z) + \frac{i}{2}\bar{\theta}^{\dot{\alpha}}\sigma^{m}_{\alpha\dot{\alpha}}\partial X_{m}(z) - \frac{1}{4}(\bar{\theta})^{2}\partial\theta_{\alpha}(z) + \frac{1}{8}\theta_{\alpha}\partial(\bar{\theta})^{2}(z) \tag{2.5}
$$

is the current associated to the $N = 1$, $D = 4$ supersymmetric covariant derivative D_{α}.

In the equations (2.4) we have introduced a notation suitable for writing the superstring field theory action we are going to deal with in section 4, labelling each term in the operators G and $\tilde{G}$ by a subscript denoting its C-charge, defined by

$$
C = \frac{1}{3}\int \Gamma^{+j}\Gamma^{-j}\,. \tag{2.6}
$$

This charge is related to the ghost number and ρ-charge, as it is obvious from the expression of J_{gh} in eq. (2.4). A detailed account of the construction of the superstring action and the role of the C-charge in this construction may be found in reference [5].

As we will discuss in detail on the next section about vertex operators, there are two sectors in the hybrid approach of open superstring.

The sector with periodic boundary conditions in the fermionic variables (θ, p, $\bar{\theta}$, $\bar{p}$) is identified with the $GSO(+)$ sector of the open superstring. This sector has manifest $N = 1$, $D = 4$ spacetime supersymmetry and was studied in a series of papers by Berkovits [3]. It is remarkable that this approach contains the four dimensional contribution of both the $NS(+)$ and $R(+)$ sub-sectors through vertex operators that are explicitly contained in $N = 1$, $D = 4$ spacetime superfields. This fact is an important advantage compared with the RNS approach where this is not possible.

The other sector is defined by anti-periodic boundary conditions in all the fermionic variables (θ, p, $\bar{\theta}$, $\bar{p}$) is identified here with the $GSO(-)$ sector of the open superstring. Since there is no fermionic zero modes in this sector the $N = 1$, $D = 4$ spacetime supersymmetry is completely broken. Furthermore, as we will show in detail, this sector contain the four-dimensional contribution of both the $NS(-)$ and $R(-)$ sub-sectors.

2.2.1 Hybrid spin field operator

In order to construct the anti-periodic sector of the fermionic hybrid variables $\theta,\ \bar{\theta},\ p,\bar{p}$ we need a spin field operator. Firstly, we have to bosonize these variables as follow:

$$\begin{array}{llll} \theta^1 = e^{\sigma_1}\,, & \theta^2 = e^{\sigma_2}\,, & \bar{\theta}^1 = e^{\bar{\sigma}_1}\,, & \bar{\theta}^2 = e^{\bar{\sigma}_2}\,, \\ p_1 = e^{-\sigma_1}\,, & p_2 = e^{-\sigma_2}\,, & \bar{p}_1 = e^{-\bar{\sigma}_1}\,, & \bar{p}_2 = e^{-\bar{\sigma}_2}\,, \end{array}$$

where the chiral bosons $\sigma_1,\ \sigma_2,\ \bar{\sigma}_2,\ \bar{\sigma}_1$ satisfy the OPEs:

$$\begin{aligned} \sigma_1(z)\sigma_1(w) &= \ln(z-w)\,, \qquad \sigma_2(z)\sigma_2(w) = \ln(z-w)\,, \\ \bar{\sigma}_1(z)\bar{\sigma}_1(w) &= \ln(z-w)\,, \qquad \bar{\sigma}_2(z)\bar{\sigma}_2(w) = \ln(z-w)\,. \end{aligned} \tag{2.8}$$

In general, all vertex operators of the $GSO(-)$ sector must contain the following general hybrid spin field

$$\Lambda^{A_1A_2B_1B_2} = e^{A_1\sigma_1+A_2\sigma_2+B_1\bar{\sigma}_1+B_2\bar{\sigma}_2}\,.$$

where A_1, A_2, B_1, B_2 are all half integer numbers and the conformal weight is

$$W\left[\Lambda^{A_1A_2B_1B_2}\right] = \frac{1}{2}[A_1(A_1-1)+A_2(A_2-1)+B_1(B_1-1)+B_2(B_2-1)]\,. \tag{2.9}$$

We will use as shorthand index notation $(+=\frac{1}{2})$, $(-=-\frac{1}{2})$, $(\oplus=\frac{3}{2})$, $(\ominus=-\frac{3}{2})$, $(\boxplus=\frac{5}{2})$, $(\boxminus=-\frac{5}{2})$. This construction is in some sense analogous to the construction of the Ramond vacuum in the RNS superstring theory [4]. Here we define $|\Lambda\rangle$ as the vacuum state for the anti-periodic variables $p\theta$, $\bar{p}\bar{\theta}$

$$|\Lambda\rangle = \Lambda^{++++}|0\rangle =: e^{(\frac{1}{2}\sigma_1+\frac{1}{2}\sigma_2+\frac{1}{2}\bar{\sigma}_1+\frac{1}{2}\bar{\sigma}_2)}(0) : |0\rangle\,. \tag{2.10}$$

This $|\Lambda\rangle$ vacuum is non degenerate and it is annihilated by all fermionic oscillators with positive modes. $\theta^\alpha_r|\Lambda\rangle = p^\alpha_r|\Lambda\rangle = 0$, $r=\frac{1}{2},\frac{3}{2},\frac{5}{2},\cdots$ etc.

2.3 Hybrid variables vs RNS variables mapping

The hybrid variables are related with the standard RNS variables by the following field redefinition [3]:

$$\begin{aligned} X^m &= e^{R+\frac{1}{2}U}(x^m)e^{-R-\frac{1}{2}U}\,, \\ \theta^\alpha &= e^{\frac{\phi}{2}}\Sigma^\alpha e^{-\frac{H}{2}}\,, \\ \bar{\theta}^{\dot\alpha} &= c\xi e^{-\frac{3}{2}\phi}\Sigma^{\dot\alpha}e^{\frac{H}{2}}\,, \\ p_\alpha &= e^{R+\frac{U}{2}}(e^{-\frac{\phi}{2}}\Sigma_\alpha e^{\frac{H}{2}})e^{-R-\frac{1}{2}U}\,, \\ \bar{p}_{\dot\alpha} &= e^{R+\frac{1}{2}U}(b\eta e^{\frac{3\phi}{2}}\Sigma_{\dot\alpha}e^{-\frac{H}{2}})e^{-R-\frac{1}{2}U}\,, \\ \partial\rho &= 3\partial\phi - cb - 2\xi\eta - \partial H\,, \\ X^{+j} &= e^{R+\frac{1}{2}U}(x^{+j})e^{-R-\frac{1}{2}U}\,, \\ X^{-j} &= e^{R+\frac{1}{2}U}(x^{-j})e^{-R-\frac{1}{2}U}\,, \\ \Gamma^{+j} &= e^{R+\frac{1}{2}U}(\xi e^{-\phi}\psi^{+j})e^{-R-\frac{1}{2}U}\,, \\ \Gamma^{-j} &= e^{R+\frac{1}{2}U}(\eta e^{\phi}\psi^{-j})e^{-R-\frac{1}{2}U}\,, \end{aligned} \tag{2.11}$$

where

$$R = \int dz \; c\xi e^{-\phi} i\psi^{-j} \partial x^{+j} \,,$$
$$U = \int dz \left[c\xi \; e^{-\phi} i\psi^{m} \partial x_{m} + \frac{1}{2}(\partial\phi + \partial H) c\partial c\xi\partial\xi e^{-2\phi} \right] .$$

This mapping ensures that eq. (2.3) are the complete set of non-vanishing OPEs of the hybrid variables.

2.4 Conditions for physical vertex operator

String fields are vertex operators acting on the Hilbert space of the string. In this section we are going to show the properties of these operators, stressing what conditions they have to satisfy to be physical.

As usual in string theory, we have several different ways of stating these physical conditions. Even though they are seemingly different, they are all equivalent. We are going to present three different possibilities. In a later section, the second and third set of conditions will be used and shown to yield the same result coming from the RNS conditions. We also argue on the equivalence of the different approaches.

Vertex operators in RNS formalism. A physical RNS vertex V has the following features:

- V is in the cohomology of the BRST operator Q, that is:

  $$\{Q, V\} = 0 \,, \qquad V \neq Q\Lambda \,.$$

  Note that V and Λ are fields in the small Hilbert space (which does not contain ξ zero modes).

- Each vertex V comes from a definite GSO sector, either GSO(+) or GSO(−). The former being $NS+$ (fermionic) and $R+$ (bosonic), the latter being $NS-$ (bosonic) and $R-$ (fermionic).

- The vertex V must have ghost number +1.

- V must have a definite picture. Usually the picture is chosen to be -1 for the $NS+$ sector and $-\frac{1}{2}$ for the $R+$ sector.

In the hybrid formalism we have two setups to consider in order to understand physical states, one coming from the $N = 2$ superalgebra and the other from the $N = 4$ superalgebra.

The conditions for the $N = 4$ superalgebra are easier to understand since the conditions may be translated directly from the RNS formalism.

$N = 4$ **physical vertex operators.** In order to understand the equivalence between these conditions and the RNS conditions, remember that the operators G^+ and $\tilde{G}^+$ are the BRST-current and the η ghost respectively. So, we may directly translate the conditions for the vertices to be in the small Hilbert space and the BRST conditions as the set of equations $\tilde{G}^+V = 0$, $G^+V = 0$. Solving the first equation (due to the triviality of the cohomology of η) we have $V = \tilde{G}^+\Phi$, where $\Phi = \xi V$. G^+ constraint yields the correct conditions. The only subtlety is the gauge invariance, which can be understood in the same way. The conditions on statistics are reversed due to the presence of the ξ fields in the vertices. Finally, picture is related to the choice of ρ-charge by the relation $P = \frac{1}{2\pi}\int dz(i\partial\rho - \frac{1}{2}p_\alpha\theta^\alpha + \frac{1}{2}\bar{p}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}})$. We refer the reader to the reference [5] for a discussion of this choice and the need for three different values for the picture number P and rho charge ρ. So, the condition for $N = 4$ physical vertices are:

- We should consider vertices in the large Hilbert space, $\Phi = \xi V$. For these kind of vertices the BRST condition of the RNS translates to:

$$G^+\tilde{G}^+\Phi = 0\,, \qquad \Phi \neq G^+\Lambda + \tilde{G}^+\tilde{\Lambda}\,.$$

  These are also the linearized equations of motion coming from the superstring field theory action.

- Again the vertices come from one of the two GSO sectors, but their statistics are now reversed with respect to the RNS.

- Ghost charge is the U(1) charge of the $N = 4$ formalism, and we should demand all the vertices to have zero ghost charge.

- Choosing a picture in a RNS formalism is the responsible for the lack of manifest supersymmetry. In the hybrid variables we may choose the values of the ρ charge instead. We choose ρ charge zero for the four-dimensional fields and ρ-charge ± 1 for the internal 6-dimensional fields. Another equivalent possibility is to choose the value of the C-charge (defined in eq. (2.6) to have one of three values in the set $\{-1/3, 0, 1/3\}$.

One should note that both set of conditions above allow us to gauge transform the vertex operators. The next set of conditions will be cast in a gauge-fixed form.

$N = 2$ **formalism.** In this formalism we need to demand the fields to be $N = 2$ superconformal primary fields with zero conformal weight and zero U(1)-charge, which mean that they have to satisfy the following conditions:

1. V must have zero conformal weight; the residue of double pole (and higher) in the OPE of L and V must be zero.

2. V must satisfy the G^+ constraint; the residue of double pole (and higher) in the OPE of G^+ and V must be zero.

3. V must satisfy the G^- constraint; the residue of double pole and (higher)in the OPE of G^- and V must be zero.

4. V must have zero ghost charge; the residue of simple poles (and higher) in the OPE of J_{gh} and V must be zero.

The issues about GSO projection and the choice of ρ charge are the same as in the $N = 4$ formalism.

In order to understand the above conditions, we must remark that the physical vertices, as usual, should be the operators in the cohomology of the $N = 2$ BRST charge which is constructed using the superconformal generators (L, G^+, G^-, J) plus four systems of ghosts, for reparametrization, two dimensional $N = 2$ SUSY and U(1) charge. However, in analogy to what happens in the RNS formalism, one may show that $N = 2$ superconformal primary fields with zero weight and zero charge are in the cohomology of $Q_{N=2}$. This is done by considering a particular explicit dependence of the ghosts for the operators, which, just like in the RNS formalism, amounts to choosing a kind of Feynmann-Siegel gauge and imposing the vertices to be annihilated by the ghosts b and $\tilde{b}$, for reparametrization and U(1) charge [6].

The next step to show the equivalence between the $N = 2$ and the RNS description is to show that the two cohomologies agree. This was done in reference [7] by showing, via a similarity transform, that the $N = 2$ BRST charge may be written as a sum of the $N = 1$ BRST charge, a topological BRST charge and the U(1) charge, all nilpotent and anticommuting with each other. The conditions that the two last BRST charges impose are just constraints in the ghost-dependence done in such a way that we can construct the vertices without the need of the $N = 2$ ghosts, using only the $N = 1$ matter and the $N = 1$ ghosts.

2.5 Vertex operators

In this subsection we present the vertex operators for the two first mass levels, both in the RNS and in the hybrid variables. The RNS vertices are easily found in the literature [8], and we may translate them to the hybrid formalism using the mapping discussed in the beginning of this section.

The first thing we are going to do is to show the results in table 1, in order to assure the important results to be all collected in a single place. After this, we explain the notation and make a few comments.

The first column denotes the different string fields, labeled by their C-charges as defined in eq. (2.6). The second and third columns show the expression for the different fields in RNS and in Hybrid variables, respectively. In order to understand how the fields are split one should have in mind the definitions for the world-sheet fields [given in section (2.1) and (2.2)] and note that the spectrum of the superstring (with both GSO sectors) in the two lowest mass-levels contains a tachyon, a gauge field and two massless fermions of opposite chiralities. Since we want to consider fields whose 4-dimensional transformations are explicit, this means that we are considering a subgroup of the full 10-dimensional

Vertex	RNS	Hybrid	W	S	GSO
Φ_0	$c\partial c\xi\partial\xi e^{-\frac{5}{2}\phi}\Sigma^{\alpha}e^{H/2}\chi^{+}_{\alpha}$	$\theta^{\alpha}(\bar{\theta})^2\chi^{+}_{\alpha}$	0	Ff	$+$
	$c\xi e^{-\frac{1}{2}\phi}\Sigma^{\dot{\alpha}}e^{-H/2}\bar{\chi}^{+}_{\dot{\alpha}}$	$\bar{\theta}^{\dot{\alpha}}(\theta)^2\bar{\chi}^{+}_{\dot{\alpha}}$	0	Ff	$+$
	$c\xi e^{-\phi}\psi^m A_m$	$\theta^{\alpha}\sigma^{m}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}A_m$	0	Bb	$+$
	$c\xi e^{-\phi}t$	Λt	$-\frac{1}{2}$	fb	$-$
	$z^{\frac{1}{2}}c\xi e^{-\frac{1}{2}\phi}\Sigma^{\alpha}e^{-H/2}\chi^{-}_{\alpha}$	$\theta^{\alpha}\Lambda\chi^{-}_{\alpha}$	0	Bf	$-$
	$z^{\frac{1}{2}}c\partial c\xi\partial\xi e^{-\frac{5}{2}\phi}\Sigma^{\dot{\alpha}}e^{H/2}\bar{\chi}^{-}_{\dot{\alpha}}$	$\bar{\theta}^{\dot{\alpha}}\Lambda\bar{\chi}^{-}_{\dot{\alpha}}$	0	Bf	$-$
$\Psi_{\frac{1}{3}}$	$c\xi e^{-\frac{1}{2}\phi}\Sigma^{\alpha}\Xi^{+j}\lambda^{+}_{+j\alpha}$	$e^{\rho}\Gamma^{+j}\theta^{\alpha}(\bar{\theta})^2\lambda^{+}_{+j\alpha}$	0	Ff	$+$
	$c\xi e^{-\phi}\psi^{-j}A_{-j}$	$e^{\rho}\Gamma^{+j}(\bar{\theta})^2 A_{-j}$	0	Bb	$+$
	$z^{\frac{1}{2}}c\xi e^{-\frac{1}{2}\phi}\Sigma^{\dot{\alpha}}\Xi^{+j}\bar{\lambda}^{-}_{+j\dot{\alpha}}$	$e^{\rho}\Gamma^{+j}\bar{\theta}^{\dot{\alpha}}\Lambda\bar{\lambda}^{-}_{+j\dot{\alpha}}$	0	Bf	$-$
$\bar{\Psi}_{-\frac{1}{3}}$	$c\partial c\xi\partial\xi e^{-\frac{5}{2}\phi}\Sigma^{\dot{\alpha}}\Xi^{-j}\bar{\lambda}^{+}_{-j\dot{\alpha}}$	$e^{-\rho}\Gamma^{-j}\bar{\theta}^{\dot{\alpha}}(\theta)^2\bar{\lambda}^{+}_{-j\dot{\alpha}}$	0	Ff	$+$
	$c\xi e^{-\phi}\psi^{+j}A_{+j}$	$e^{-\rho}\Gamma^{-j}(\theta)^2 A_{-j}$	0	Bb	$+$
	$z^{\frac{1}{2}}c\partial c\xi\partial\xi e^{-\frac{5}{2}\phi}\Sigma^{\alpha}\Xi^{-j}\lambda^{-}_{-j\alpha}$	$e^{-\rho}\Gamma^{-j}\theta^{\alpha}\Lambda\lambda^{-}_{-j\alpha}$	0	Bf	$-$

Table 1: Vertex Operators corresponding to the tachyon and the massless fields.

Lorentz group:

$$\begin{aligned} &\mathrm{SO}(9,1) \to \mathrm{SO}(3,1)\times\mathrm{SO}(6)\,, \\ &\mathrm{SO}(9,1) \to \mathrm{SO}(3,1)\times\mathrm{SU}(3)\times\mathrm{U}(1)\,. \end{aligned} \tag{2.12}$$

The last line comes from the fact that we need to consider spinors. The representations of the $D = 10$ Lorentz algebra decompose as:

$$\begin{array}{ccccccc} 10 & \to & 4 & \oplus & 3 & \oplus & \bar{3} \\ A_\mu & & A_m & & A_{-i} & & A_{+i} \end{array}$$

$$\begin{array}{cccccccc} 16 & \to & (2,1_{+3}) & \oplus\ (2,3_{-1}) & \oplus & (2',1_{-3}) & \oplus & (2',\bar{3}_{+1}) \\ \chi^{+}_{a} & & \chi^{+}_{\alpha} & \lambda^{+}_{-i\alpha} & & \bar{\chi}^{+}_{\dot{\alpha}} & & \bar{\lambda}^{+}_{+i\dot{\alpha}} \end{array}$$

$$\begin{array}{cccccccc} 16' & \to & (2',1_{+3}) & \oplus\ (2',3_{-1}) & \oplus & (2,1_{-3}) & \oplus & (2,\bar{3}_{+1}) \\ \chi^{-}_{a} & & \bar{\chi}^{-}_{\dot{\alpha}} & \bar{\lambda}^{-}_{-i\dot{\alpha}} & & \chi^{-}_{\alpha} & & \lambda^{-}_{+i\alpha}\,. \end{array}$$

In the above equations, primed quantities indicate anti-weyl representations of the appropriate Lorentz group and unprimed represent the weyl representations. The number in parenthesis mean the dimension of the representations and the subscript is the U(1) charge. Below the representation we indicate the fields which transform under them. With this splitting, one can understand the origin of all the spacetime fields appearing in the table 1.

Also on the table 1, the W column shows the conformal weight of the fields. The S column represents both the statistics of the spacetime fields, b stands for spacetime boson and f for spacetime fermionic; the capital letter denote the statistics in the world-sheetfield that means: B stands for world sheet bosonic field and F stands for world sheet fermionic field. Finally, the last column represents the GSO sector originating the fields.

In the next section we are going to consider the construction of $GSO(-)$ vertices directly in the hybrid formalism, checking the physical conditions described earlier. In the second part of the paper we are going to use these vertices to compute the superstring field theory action, collecting the $GSO(+)$ part in superfields.

3. Physical $GSO(-)$ vertex operators in hybrid formalism

In the $GSO(-)$ sector the space time supersymmetry is broken, however we can still use the set of hybrid variables to explicitly construct vertex operators. In particular we have built the physical vertices for the tachyon and the massless fermion by using only the hybrid variables, showing the appropriate physical conditions to be satisfied. Most of the calculations are exhibited explicitly, since they are useful throughout this work.

3.1 Φ^- vertex operator

In this subsection we discuss the construction of C-neutral vertices that are independent of the detailed structure of the compact manifold. As seen in the previous section, we should have a tachyon T and the massless fermions χ^- in this sector.

3.1.1 Tachyon vertex

There is a unique spin operator with the lowest conformal weight ($W = -\frac{1}{2}$) Λ^{++++}. This is the candidate to represent the part of zero momentum of the tachyon; however, it must be verified that all physical conditions for this vertex are indeed satisfied.

Let us consider the candidate for the tachyon vertex

$$\Lambda^{++++}T(X)\,. \tag{3.1}$$

We can show that this vertex is fermionic and it is exactly the zero momentum part of the RNS tachyon vertex in the large Hilbert space $c\xi e^{-\phi}$. This vertex corresponds to the lowest level NS sector that is projected out by the GSO projection of superstring.

Given the vertex above for the Tachyon, it is trivial to verify that the zero ghost charge and zero C-charge conditions are satisfied. We still need to check the conditions for the OPEs with T and G.

For simplicity we consider the tachyon vertex with definite momentum $T(k,z) = \Lambda^{++++}e^{ikX}$ (where kX stands for $k_mX^m + k^{+i}X^{-i} + k^{-i}X^{+i}$) and we are going to check the physical conditions for this vertex.

$N = 2$ conditions. Using the stress tensor we get

$$\begin{aligned} L(z)T(k,0) &= \frac{(-\frac{1}{2}+\frac{k^2}{2})}{z^2}T(k,0) + \frac{1}{z}\partial T(k,0) + \cdots\,, \\ k^2 &= k_mk^m + k^{+i}k^{-i}\,, \end{aligned}$$

requiring that the double pole vanishes we get the on-shell condition for the momentum k of the tachyon $k^2 = 1$.

Now we need to verify the G constraints, ensuring the vanishing of double poles and higher.

We find that the OPE $G^+(z)T(k,w)$ is

$$\begin{aligned}(e^\rho(d)^2+\Gamma^{-j}\partial X^{+j})(z)\Lambda^{++++}e^{ikX}(w) \;=\; & \frac{1}{(z-w)}\{[-2\Lambda^{--++}+k_m(p\sigma^m\bar\theta\Lambda^{++++}) \\ & +\frac{1}{2}(k^mk_m-1)\Lambda^{++\oplus\oplus}]e^\rho \\ & +k^{+j}\Gamma^{-j}\Lambda^{++++}\}e^{ikX}+\cdots .\end{aligned}$$

The OPE $G^-(z)T(k,w)$ is

$$\begin{aligned}(e^{-\rho}(\bar d)^2+\Gamma^{+j}\partial X^{-j})(z)\Lambda^{++++}e^{ikX}(w) \;=\; & \frac{1}{(z-w)}\{[-2\Lambda^{++--}+k_m(\bar p\bar\sigma^m\theta\Lambda^{++++})+ \\ & +\frac{1}{2}(k^mk_m+1)\Lambda^{\oplus\oplus--}]e^{-\rho}+ \\ & +k^{+j}\Gamma^{-j}\Lambda^{++++}\}e^{ikX}+\cdots\end{aligned}$$

where the notation $(p\sigma\theta\Lambda)$ describes only the indices structure in the spin field, no contractions implied. We see immediately that there are no double poles in these OPEs, which implies that the vertex satisfies the physical conditions and no polarization conditions are imposed in the momentum k. We have shown that the tachyon vertex $T(k,z)$ is physical.

Now we proceed by showing that the $N=4$ physical conditions are also satisfied. The only non-trivial part to be shown is that the equation of motion is indeed satisfied.

Equation of motion. The gauge invariant Equation of motion is $[\tilde G^+]_0[G^+]_0T(k,z)=0$ we first use our result for the OPE in $G^+(z)T(k,w)$ (the notation $[\mathcal{O}]_nV$ means the residue of the pole of $(n+d)$ order in the OPE between $\mathcal{O}$ and V, where d is the conformal weight of the operator $(\mathcal{O})$).

Using the results above, we need to compute the following OPEs (for the first piece of the operator $\tilde G^+$ the relevant terms are the cubic poles, for the second piece the relevant terms are the double poles, this is due to the different ρ dependence in each term):

$$\begin{aligned}(\bar d)^2(z)\Lambda^{--++}e^{ikX}(z) &= \frac{1}{(z-w)^3}(k^mk_m)\Lambda^{++++}e^{ikX}(w)+\ldots \\ (\bar d)^2(z)(p\bar\theta\Lambda^{++++})e^{ikX}(w) &= \frac{\bar\sigma^m}{(z-w)^3}k_m\Lambda^{++++}e^{ikX}(w)+\ldots \\ (\bar d)^2(z)\Lambda^{++\oplus\oplus}e^{ikX}(w) &= (-2)\frac{1}{(z-w)^3}\Lambda^{++++}e^{ikX}(w)+\ldots, \\ \partial X^{-k}(z)k^{+j}e^{ikX} &= \frac{1}{(z-w)^2}k^{+j}k^{-j}+\ldots\end{aligned}$$

with this OPEs we readily see that

$$\left[\tilde G^+\right]_0[G^+]_0T(k,z)=(k^2-1)T(k,z)=0 \tag{3.2}$$

which is the correct equation of motion for the tachyon in momentum representation.

3.1.2 Massless fermion vertices

On the next level of the spectrum of states, we have two pairs of spin field vertices with zero conformal weight ($W = 0$). It can be shown that each pair of vertex is bosonic. These vertex are candidates to be the zero momentum contribution associated with the massless space-time fermions.

The OPEs between the hybrid variables and hybrid spin fields have a square root branch cut, from this OPE we may find the excited spin fields we are looking for. We may consider both p and θ (with their conjugate fields) to construct the spin fields, which indeed have the correct conformal weight:

$$\begin{aligned}
\Theta_\alpha &\equiv \, :\theta_\alpha \Lambda^{++++} := \Lambda^{\overset{\oplus}{+}\overset{+}{\oplus}++}\,, \\
\bar{\Theta}_{\dot{\alpha}} &\equiv \, :\bar{\theta}_{\dot{\alpha}} \Lambda^{++++} := \Lambda^{++\overset{\oplus}{+}\overset{+}{\oplus}}\,, \\
\Pi_\alpha &\equiv \, :p_\alpha \Lambda^{++++} := \Lambda^{\overset{-}{+}\overset{+}{-}++}\,, \\
\bar{\Pi}_{\dot{\alpha}} &\equiv \, :\bar{p}_{\dot{\alpha}} \Lambda^{++++} := \Lambda^{++\overset{-}{+}\overset{+}{-}}\,.
\end{aligned} \tag{3.3}$$

Using these fields we may construct a fermion vertex for the massless (4-dimensional) sector:

$$\Phi = (\Theta_\alpha \chi_1^\alpha + \Pi_\alpha \chi_2^\alpha + \bar{\Theta}^{\dot{\alpha}} \bar{\chi}_{1\dot{\alpha}} + \bar{\Pi}^{\dot{\alpha}} \bar{\chi}_{2\dot{\alpha}}) e^{ikX} \tag{3.4}$$

where we have decided to work in momentum representation and the χ-fields represent the polarizations of the fermions. We assume that the momentum k has only components in the four dimensional space time in order to avoid the mixing with the other fields. (Remember that we have a fermion in 10-dimensions and its equation of motion should be a Dirac equation in 10-dimensions, due to the Lorentz symmetry breaking pattern we are using, the different fermion vertices in the hybrid formalism will mix with each other, unless we assume this simplifying condition.)

We are now going to check whether this vertex satisfies the physical condition or not:

$N = 2$ constraints. The vanishing of the double poles (and higher) in the OPE of the stress tensor with the vertex operator is quite trivial since only the first term in eq. (2.4) contributes to the double pole, yielding the mass-shell condition $k^2 = 0$ if this vertex is to have zero conformal weight.

The G constraints are a bit more subtle and should give us polarization conditions. Let us explicitly show the relevant OPEs and discuss it:

$$\begin{aligned}
G^+(z)(\Pi\chi_2)(0) &\sim \frac{e^\rho}{z}\left[k^2 \Lambda^{\overset{-}{+}\overset{+}{-}\oplus\oplus}\chi_2 + k_m \chi_2 \sigma^m \Lambda^{--\overset{\oplus}{+}\overset{+}{\oplus}}\right] + \cdots \\
G^+(z)(\Theta\chi_1)(0) &\sim \frac{e^\rho}{z^2}\left[\Pi\chi_1 + k_m \chi_1 \sigma^m \bar{\Theta}\right] + \frac{e^\rho}{z}\left[k^2 \Lambda^{\overset{\oplus}{+}\overset{+}{\oplus}\oplus\oplus}\chi_1\right] + \cdots \\
G^+(z)(\bar{\Pi}\bar{\chi}_2)(0) &\sim \frac{e^\rho}{z^2}\left[k^2 \bar{\Theta}\bar{\chi}_2 + k_m \bar{\chi}_2 \bar{\sigma}^m \Pi\right] + \frac{e^\rho}{z}\left[\Lambda^{--\overset{-}{+}\overset{+}{-}}\bar{\chi}_2 + k_m \bar{\chi}_2 \sigma^m \Lambda^{\overset{\oplus}{+}\overset{+}{\oplus}\oplus\oplus}\right] + \cdots \\
G^+(z)(\bar{\Theta}\bar{\chi}_1)(0) &\sim \frac{e^\rho}{z}\left[k_m \bar{\chi}_1 \bar{\sigma}^m \Lambda^{\overset{-}{+}\overset{+}{-}\oplus\oplus} + \Lambda^{--\overset{\oplus}{+}\overset{+}{\oplus}}\bar{\chi}_1\right] + \cdots
\end{aligned} \tag{3.5}$$

where we omitted the momentum dependence and contraction in fermionic indices. The G^- constraints have the same form and can be read from this expression, changing the ρ dependence to $e^{-\rho}$ and moving the bars to appropriate places (for instance $G^-\Pi\chi_2 = \overline{G^+\bar{\Pi}\bar{\chi}_2}$).

Vanishing of the double poles in this OPEs imply the following conditions for the polarizations:

$$\begin{aligned} \chi_1 &= k_m \sigma^m \bar{\chi}_2 \,, \\ \bar{\chi}_1 &= k_m \bar{\sigma}^m \chi_2 \,. \end{aligned}$$

A physical vertex is a superconformal primary state defined modulo spurious states (with zero norm), created by the negative frequency part of the superconformal generators. So, we need to identify physical states modulo gauge transformations $\delta\Phi = G^+(\bar{\Pi}\bar{\zeta} + \Theta k_m \sigma^m \bar{\zeta}) + G^-(\Pi\zeta + \bar{\Theta} k_m \bar{\sigma}^m \zeta)$, since the transformation should be of the form $\delta\Phi = G\Lambda$, with Λ physical. This identification yields the following gauge transformations for the vertices:

$$\begin{aligned} \delta\bar{\chi}_1 &= k^2 \bar{\zeta} \\ \delta\chi_2 &= k_m \sigma^m \bar{\zeta} \,, \end{aligned}$$

and analogous transformations for the other polarizations. So we realize that not all the spacetime fields in eq. (3.4) are independent. This is the same situation we encounter in the $GSO(+)$ sector when considering an $N = 1, D = 4$ vector superfield as vertex operator for the massless sector [3]. This way we see that we have only two spacetime massless spinor fields in this sector, with opposite chiralities.

Now we should check the $N = 4$ conditions for this vertex:

$N = 4$ **formalism.** The $N = 2$ formalism gives us a gauge fixed vertex operator, while in the $N = 4$ formalism we have an equation of motion for the vertex operator, as well as a gauge invariance.

The equation of motion is obtained by $\tilde{G}^+G^+[\Phi] = 0$, where we are going to use the same vertex eq. (3.4) as before and G applied to the vertex means the contour integral, as usual. Since we have already calculated the OPEs eq. (3.5), we may use this result taking care of the pole structure coming also from the ρ-dependence. We may see that:

$$\begin{aligned} \tilde{G}^+G^+(\Theta\chi_1) &= k_m \chi_1 \sigma^m \bar{\Pi} + k^2 \Theta \chi_1 \,, \\ \tilde{G}^+G^+(\bar{\Theta}\bar{\chi}_1) &= k_m \bar{\chi}_1 \bar{\sigma}^m \Pi + k^2 \bar{\Theta} \bar{\chi}_1 \,, \\ \tilde{G}^+G^+(\Pi\chi_2) &= k^2 k_m \chi_2 \sigma^m \bar{\Theta} + k^2 \Pi \chi_2 \,, \\ \tilde{G}^+G^+(\bar{\Pi}\bar{\chi}_2) &= k^2 k_m \bar{\chi}_2 \bar{\sigma}^m \Theta + k^2 \bar{\Pi} \bar{\chi}_2 \,. \end{aligned} \tag{3.6}$$

and the equation of motion imply:

$$\begin{aligned} k_m \chi_1 \sigma_m + k^2 \bar{\chi}_2 &= 0 \,, \\ k_m \bar{\chi}_1 \bar{\sigma}_m + k^2 \chi_2 &= 0 \,, \\ k^2 (k_m \bar{\chi}_2 \bar{\sigma}_m + \chi_1) &= 0 \,, \\ k^2 (k_m \chi_2 \sigma_m + \bar{\chi}_1) &= 0 \,. \end{aligned}$$

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The gauge invariances, as described in the section on physical conditions, are:

$$\begin{aligned} G^+\left[e^{-\rho}\left(\Theta\lambda_1+\bar{\Pi}\bar{\lambda}_2\right)\right] &= \Pi\lambda_1+k_m\lambda_1\sigma^m\bar{\Theta}+k^2\bar{\Theta}\bar{\lambda}_2+k_m\bar{\lambda}_2\bar{\sigma}^m\Pi\,,\\ \tilde{G}^+\left[e^{2\rho}\Gamma^{+i}\Gamma^{+j}\Gamma^{+k}\left(\Pi\lambda_2+\bar{\Theta}\bar{\lambda}_1\right)\right] &= \bar{\Pi}\bar{\lambda}_1+k_m\bar{\lambda}_1\bar{\sigma}^m\Theta+k^2\Theta\lambda_2+k_m\lambda_2\sigma^m\bar{\Pi}\,, \end{aligned}$$

from this conditions we may read the gauge transformations for the spacetime fields:

$$\begin{aligned} \delta\chi_1 &= k_m\bar{\lambda}_1\bar{\sigma}^m+k^2\lambda_2\,,\\ \delta\bar{\chi}_1 &= k_m\lambda_1\sigma^m+k^2\bar{\lambda}_2\,,\\ \delta\chi_2 &= \lambda_1+k_m\bar{\lambda}_2\bar{\sigma}^m\,,\\ \delta\bar{\chi}_2 &= \bar{\lambda}_1+k_m\lambda_2\sigma^m\,. \end{aligned}$$

All this is in complete analogy with the $GSO(+)$ sector, again we have only two massless spinor spacetime fields with opposite chiralities, the usual equation of motion is verified. What amounts to choose the Wess-Zumino Gauge in the $GSO(+)$ sector is to choose the spin fields Θ and $\bar{\Theta}$ to be in the vertex operator, this is what we have done in table 1.

The equivalence between the two formalisms described above is simple, the $N=2$ formalism gives us a gauge-fixed vertex operator, whose gauge fixing conditions are exactly the G constraints.

3.2 Ψ^- and $\bar{\Psi}^-$ vertex operators

In this subsection we construct the vertices that dependent of the structure of the compact manifold. We represent these vertices as Ψ^- and $\bar{\Psi}^-$ with C-charge $1/3$ and $-1/3$ respectively. Since these two vertices are conjugate to each other we should treat them together.

The candidates for a vertex of this kind are:

$$\begin{aligned} \Psi(k,z) &= e^{\rho}\Gamma^{+i}\left(\Theta\chi_1+\Pi\chi_2+\bar{\Theta}\bar{\eta}_1+\bar{\Pi}\bar{\eta}_2\right)e^{ikX}\,,\\ \bar{\Psi}(k,z) &= e^{-\rho}\Gamma^{-i}\left(\Theta\eta_1+\Pi\eta_2+\bar{\Theta}\bar{\chi}_1+\bar{\Pi}\bar{\chi}_2\right)e^{ikX}\,. \end{aligned}$$

The necessary OPEs were already calculated in eq. (3.5), but the relevant poles are different due to the contribution of ρ and Γ. (Note also that we are still considering the momentum running only in 4 dimensions). Let us proceed by checking the physical conditions:

$N=2$ **formalism.** Vanishing of the double pole in the OPE of the stress tensor L and the vertex operator is just like before. Only the first term and L contributes and the physical condition is the mass-shell condition $k^2=0$.

In order to check the G^+ constraint in the Ψ vertex one should note that the order of the poles change, but the terms are maintained in the equation eq. (3.5). Besides the ρ and Γ dependence, the poles of this equation acquire one more negative power. For the G^- constrain we have something analogous, but the poles acquire one more positive power, since the higher order poles were second order, and we should ensure the vanishing of all second and higher order poles, we see that G^- puts no constraint in Ψ, since there are no double poles (or higher) in this OPE. However, the G^+ has additional constraints:

$$\begin{aligned}\bar{\eta}_2 &= 0\,,\\ \chi_1 &= -k_m\bar{\eta}_2\bar{\sigma}^m\,,\\ \bar{\eta}_1 &= k_m\chi_2\sigma^m\,,\end{aligned}$$

and we see that there is actually only one massless spacetime spinor in the vertex operator Ψ^- and since $k^2=0$, η satisfies $k_m\bar{\eta}_1\sigma^m=0$.

Same reasoning is valid to the $\bar{\Psi}$ vertex operator, and only combinations of Θ and $\bar{\Pi}$ survive.

$N=4$ **formalism.** All the necessary computations have already been done in the Φ vertex discussed above. In order to avoid repetition, we just state that we have only two physical massless spinor spacetime fields in these two vertices and we can choose (gauge-fixing) Ψ to have only the $\bar{\Theta}$ combination and $\bar{\Psi}$ to have only the Θ combination, as a result of the 8 equations of motion and the 8 gauge invariances.

With the vertex operators discussed above we are going to proceed to the study of the superstring field theory and compute the contribution of these vertices to it.

4. Hybrid superstring field theory for non-BPS D-branes

The action we are going to use is Berkovits superstring field theory action [5], which is the only string field action that allows one to include the Ramond sector in a gauge invariant way without divergence problems at tree level.

The action constructed in [5] is

$$\begin{aligned}S = &\left\langle \left(e^{-\Phi}G_{-1}e^{\Phi}\right)\left(e^{-\Phi}G_{0}e^{\Phi}\right) + \left(e^{-\Phi}G_{\frac{2}{3}}e^{\Phi}\right)\left(e^{-\Phi}G_{-\frac{1}{3}}e^{\Phi}\right)\right\rangle_D + \\ &+\left\langle \int_0^1 dt\left(e^{-\tilde{\Phi}}\partial_t e^{\tilde{\Phi}}\right)\left(\left\{e^{-\tilde{\Phi}}G_{-1}e^{\tilde{\Phi}},\, e^{-\tilde{\Phi}}G_0e^{\tilde{\Phi}}\right\} + \left\{e^{-\tilde{\Phi}}G_{-\frac{2}{3}}e^{\tilde{\Phi}},\, e^{-\tilde{\Phi}}G_{-\frac{1}{3}}e^{\tilde{\Phi}}\right\}\right)\right\rangle_D - \\ &-\left\langle e^{-\Phi}\bar{\Omega}e^{\Phi}\Omega + \bar{\Omega}e^{\Phi}G_{-\frac{2}{3}}e^{-\Phi} + \Omega e^{-\Phi}G_{-\frac{1}{3}}e^{+\Phi}\right\rangle_D - \\ &-\left\langle \frac{1}{2}\bar{\Omega}G_{-\frac{1}{3}}\bar{\Omega} + \frac{1}{3}\bar{\Omega}^3\right\rangle_{\bar{F}} + \left\langle \frac{1}{2}\Omega G_{-\frac{2}{3}}\Omega + \frac{1}{3}\Omega^3\right\rangle_F\end{aligned} \tag{4.1}$$

where the operators G appearing in this action are defined in eq. (2.4), but now with the $+$ superscript dropped in order to avoid confusion with GSO notation, subscript denotes C-charge. We should also note that there are three different correlators. The D-correlator is defined in the large Hilbert Space of the superstring. The F and $\bar{F}$ correlators are chiral (anti-chiral) subspaces defined with the trivial cohomology pieces of the G operator, G_{-1} and G_0.

The string fields appearing in the action are defined in such a way that Φ has zero C-charge, $\Omega \equiv G_{-1}\Psi$, is a chiral field obtained from the string field Ψ with C-charge 1/3 and $\bar{\Omega} \equiv G_0\bar{\Psi}$ is an anti-chiral string field obtained from a string field $\bar{\Psi}$ with C-charge $-1/3$. We are using the notation $\tilde{\Phi} = t\Phi,\ \ 0 \le t \le 1$.

Also, one should remember that the products between any two string fields are the Witten's midpoint interaction. The prescription we are going to use is:

$$\langle V_1 V_2 \cdots V_N \rangle = \left(-\frac{4i}{N}\right)^{\sum_{k=1}^{N} h_k} e^{\frac{2\pi i}{N}\sum_{l=1}^{N} h_l (l-1)} \left\langle \Pi_{j=1}^{N} V_j \left(e^{\frac{2\pi i}{N}(j-1)}\right)\right\rangle$$

where V_i here are conformal primary fields with conformal weight h_i.

The correlator of the conformal field theory fields in the right-hand side is calculated in the disk, since this prescription means that we take the N open string world-sheets, represented by N upper half-disks and map them to the unit disk (defined in global coordinates) via global conformal transformations, taking care to glue the right sides of each string. A detailed discussion of this Witten interaction may be found in [9].

In order to introduce $GSO(-)$ sector, we are going to consider the following tensor products:

$$\begin{aligned}
&\hat{\Phi} \equiv \Phi^+ \otimes I + \Phi^- \otimes \sigma_1\,, && \\
&\hat{\Psi} \equiv \Psi^+ \otimes I + \Psi^- \otimes \sigma_1\,, && \hat{\bar{\Psi}} \equiv \bar{\Psi}^+ \otimes I + \bar{\Psi}^- \otimes \sigma_1\,,\\
&\hat{\Omega} \equiv \Omega^+ \otimes \sigma_3 + \Omega^- \otimes i\sigma_2\,, && \hat{\bar{\Omega}} \equiv \bar{\Omega}^+ \otimes \sigma_3 + \bar{\Omega}^- \otimes i\sigma_2\,,
\end{aligned}$$

$$\begin{aligned}
&\hat{G}_0 \equiv G_0 \otimes \sigma_3\,, && \hat{G}_{-1} \equiv G_{-1} \otimes \sigma_3\,,\\
&\hat{G}_{-\frac{1}{3}} \equiv G_{-\frac{1}{3}} \otimes \sigma_3\,, && \hat{G}_{-\frac{2}{3}} \equiv G_{-\frac{2}{3}} \otimes \sigma_3
\end{aligned}$$

where σ_i are Pauli matrices. All this only means that we split string fields in two pieces according to the GSO sector they have come out and tensor it by appropriate internal Chan-Paton factors, since the commutative properties of the two sectors are different (GSO$(-)$ sector being Grassmann odd), this is to ensure the ciclicity of the correlators as in any usual CFT. Also the BRST-like operators need to be tensorized in such a way that, at the end we get all the properties of a derivation as they act in the string fields, in particular Leibniz rule should be verified. It is important to remark only that the (anti)chiral fields appearing in the action are split in the same way, but as they have a differential operator acting on it (G_0 or G_{-1}) we have the tensor product of σ_3 carried by the G-operator and the matrices in the string fields, either Ψ or $\bar{\Psi}$. The correlators should include the trace over these internal Chan-Paton matrices.

4.1 Vertex operators in SSFT

The vertex operator for the Tachyon and the massless sector are:

$$\begin{aligned}
\Phi^+ &= v(x,\theta,\bar{\theta})\,, & \Phi^- &= \Lambda^{++++}T(x) + \Theta^{\alpha}\chi^-_{\alpha}(x) + \bar{\Theta}^{\dot{\alpha}}\bar{\chi}^-_{\dot{\alpha}}(x)\,,\\
\Psi^+ &= e^{\rho}\Gamma^{+j}\bar{\theta}^2\omega_{+j}(x,\theta)\,, & \Psi^- &= e^{\rho}\Gamma^{+j}\bar{\Theta}^{\dot{\alpha}}\bar{\lambda}^-_{+j\dot{\alpha}}(x)\,,\\
\bar{\Psi}^+ &= e^{-\rho}\Gamma^{-j}\theta^2\bar{\omega}_{-j}(x,\bar{\theta})\,, & \bar{\Psi}^- &= e^{-\rho}\Gamma^{-j}\Lambda^{\alpha}\lambda^-_{-j\alpha}(x)\,.
\end{aligned}$$

These fields are in table 1. Here we have collected only the $GSO(+)$ string fields in $D=4, N=1$ superfields, whose component expansion are:

$$\begin{aligned}
v(x,\theta,\bar{\theta}) &= \theta\sigma^m\bar{\theta}A_m(x) + \theta\theta\bar{\theta}\bar{\chi}^+(x) + \bar{\theta}\bar{\theta}\theta\chi^+(x) + \theta\theta\bar{\theta}\bar{\theta}D(x)\,,\\
\omega_{-i}(x,\theta) &= A_{-i}(x) + \theta\lambda^+_{-i}(x) + \theta\theta F_{-i}(x)\,,\\
\bar{\omega}_{+i}(x,\bar{\theta}) &= A_{+i}(x) + \bar{\theta}\bar{\lambda}^+_{+i}(x) + \bar{\theta}\bar{\theta}\bar{F}_{+i}(x)\,.
\end{aligned}$$

The $GSO(-)$ sector is directly taken from the table 1 and written in the appropriate string fields, according to their C-charge.

We also need to calculate the (anti)chiral superstring fields that appear in the action. The idea is to pick the simple pole in the OPE of the integrand of the appropriate G with the string fields.

$$\begin{aligned}\Omega^+ &= G_{-1}\Psi^+ = \oint dw\left(\frac{1}{12}\bar{d}^{\dot\alpha}\bar{d}_{\dot\alpha}e^{-2\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}(w)\right)\left(e^{\rho}\Gamma^{+j}\omega^{-j}(x,\theta)\bar\theta^2(z)\right)\\ &= \oint dw\frac{1}{12}\frac{2}{(w-z)^2}(w-z)^2e^{-\rho}6\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\frac{1}{z}\omega^{-l}\\ &= e^{-\rho}\Gamma^{-j}\Gamma^{-k}\omega^{-l}\epsilon^{jkl}.\end{aligned} \tag{4.2}$$

$$\begin{aligned}\bar\Omega^+ &= G_0\bar\Psi^+ = \oint dz\left(\frac{1}{2}d^{\alpha}d_{\alpha}e^{\rho}(w)\right)\left(e^{-\rho}\Gamma^{-j}\bar\omega^{+j}(x,\bar\theta)\theta^2(z)\right)\\ &= \oint dw\frac{1}{2}\frac{2}{(w-z)^2}z\Gamma^{-j}\bar\omega^{+j}\\ &= \Gamma^{-j}\bar\omega^{+j}.\end{aligned} \tag{4.3}$$

$$\begin{aligned}\Omega^- &= G_{-1}\Psi^- = \oint dw\left(\frac{1}{12}\bar{d}^{\dot\alpha}\bar{d}_{\dot\alpha}e^{-2\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}(w)\right)\left(e^{\rho}\Gamma^{+i}\bar\Theta\bar\lambda^{-i}(z)\right)\\ &= \oint dw\frac{1}{12}\left((w-z)^2e^{-\rho}\right)\left(\frac{6\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\eta^{il}}{z}\right)\left(\frac{2\bar\Pi\bar\lambda^{-i}+i\partial_m\bar\lambda^{-j}\bar\sigma^m\Theta}{z^2}\right)\\ &= e^{-\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\left(\bar\Pi\bar\lambda^{-i}+\frac{i}{2}\partial_m\bar\lambda^{-j}\bar\sigma^m\Theta\right).\end{aligned} \tag{4.4}$$

$$\bar\Omega^- = G_0\bar\Psi^- = \Gamma^{-j}\left(\Pi\lambda^{-i}+\frac{i}{2}\partial_m\lambda^{+j}\sigma^m\bar\Theta\right). \tag{4.5}$$

5. Tachyon and massless sector SSFT action

We are now almost ready to calculate the superstring field theory action for the tachyon and massless sector, the only piece of information still lacking is the definition of the non-vanishing norms, i.e., a choice for the normalization of the correlators:

$$\left\langle\frac{1}{24}\theta^2\bar\theta^2e^{-\rho}\epsilon^{ijk}\Gamma^{-i}\Gamma^{-j}\Gamma^{-k}\right\rangle_D = 1\,, \tag{5.1}$$

$$\left\langle G_0\left(\frac{1}{24}\theta^2\bar\theta^2e^{-\rho}\epsilon^{ijk}\Gamma^{-i}\Gamma^{-j}\Gamma^{-k}\right)\right\rangle_{\bar F} = \left\langle\frac{1}{24}\bar\theta^2\epsilon^{ijk}\Gamma^{-i}\Gamma^{-j}\Gamma^{-k}\right\rangle_{\bar F} = 1\,, \tag{5.2}$$

$$\left\langle G_{-1}\left(\frac{1}{24}\theta^2\bar\theta^2e^{-\rho}\epsilon^{ijk}\Gamma^{-i}\Gamma^{-j}\Gamma^{-k}\right)\right\rangle_{F} = \left\langle\frac{1}{4}e^{-3\rho}\theta^2(\Gamma\cdot\partial\Gamma)^3\right\rangle = 1\,. \tag{5.3}$$

The first norm is exactly the non-vanishing norm in the large Hilbert Space of the Superstring, in RNS variables this is $\xi e^{-2\phi}c\partial c\partial^2 c$, giving us the correct background charge for all the ghost fields. Its value is related with the superconformal killing vectors of the disk as discussed in [8]. The non-vanishing norms in the (anti)chiral subspaces are obtained from the norm of the large Hilbert space by the application of the suitable differential operator (with trivial cohomology) which defines the (anti)chiral subspace.

The Superstring Field Theory Action is non-polynomial, in order to deal with it we are going to expand the exponentials and keep terms up to four string fields. For the vertices we are considering, all other terms vanish since it is impossible to write higher order terms with the string fields at first and second mass levels that cancel the background charge yielding non-vanishing correlators.

For the first part of the action we have:

$$
\begin{aligned}
S^{(1)}_{WZ} &= \left\langle \left(e^{-\hat\Phi}\hat G_{-1}e^{\hat\Phi}\right)\left(e^{-\hat\Phi}\hat G_0 e^{\hat\Phi}\right)\right\rangle_D - \left\langle \int_0^1 dt (e^{-\tilde{\hat\Phi}}\hat\partial_\tau e^{\tilde{\hat\Phi}})\{e^{-\tilde{\hat\Phi}}G_{-1}e^{\tilde{\hat\Phi}}, e^{-\tilde{\hat\Phi}}G_0 e^{\tilde{\hat\Phi}}\}\right\rangle_D \\
&= 2\sum_{M,N=0}^{\infty}\frac{(-1)^N}{(M+N+2)!}\binom{M+N}{N}\left\langle\left(\hat G_{-1}\hat\Phi\right)\hat\Phi^M\left(\hat G_0\hat\Phi\right)\hat\Phi^N\right\rangle_D \\
&= 2\Bigg[\frac{1}{2}\left\langle\left(\hat G_{-1}\hat\Phi\right)\left(\hat G_0\hat\Phi\right)\right\rangle_D - \frac{1}{6}\left\langle\hat\Phi\left\{\hat G_{-1}\hat\Phi\right\}\left\{\hat G_0\hat\Phi\right\}\right\rangle_D - \\
&\qquad - \frac{1}{24}\left\langle\left[\hat\Phi,\hat G_{-1}\hat\Phi\right]\left[\hat\Phi,\hat G_0\hat\Phi\right]\right\rangle_D + \cdots\Bigg] \\
&= \left\langle\left(\hat G_{-1}\hat\Phi\right)\left(\hat G_0\hat\Phi\right)\right\rangle_D - \frac{1}{3}\left\langle\hat\Phi\left\{\hat G_{-1}\hat\Phi\right\}\left\{\hat G_0\hat\Phi\right\}\right\rangle_D - \\
&\quad -\frac{1}{12}\left\langle\left[\hat\Phi,\hat G_{-1}\hat\Phi\right]\left[\hat\Phi,\hat G_0\hat\Phi\right]\right\rangle_D + \cdots
\end{aligned}
\tag{5.4}
$$

Splitting in $GSO(+)$ and $GSO(-)$ fields and taking care of the Pauli matrices multiplying the different string fields and operators, the relevant terms we need to compute are the following:

$$
\begin{aligned}
\left\langle\hat G_{-1}\hat\Phi,\hat G_0\hat\Phi\right\rangle_D &= \underbrace{\langle G_{-1}\Phi^+G_0\Phi^+\rangle_D}_{GSO(+)} - \langle G_{-1}\Phi^-G_0\Phi^-\rangle_D\,, \\
\left\langle-\frac{1}{3}\hat\Phi\left\{\hat G_{-1}\hat\Phi,\hat G_0\hat\Phi\right\}\right\rangle_D &= \underbrace{\left\langle\frac{1}{3}\Phi^+\left\{G_{-1}\Phi^+,G_0\Phi^+\right\}\right\rangle_D}_{GSO(+)} \\
&\quad + \left\langle\frac{1}{3}\Phi^+\left\{\hat G_{-1}\Phi^-,\hat G_0\Phi^-\right\}\right\rangle_D - \\
&\quad - \left\langle\frac{1}{3}\Phi^-\left[G_{-1}\Phi^+,G_0\Phi^-\right]\right\rangle_D + \\
&\quad + \left\langle\frac{1}{3}\Phi^-\left[G_{-1}\Phi^-,G_0\Phi^+\right]\right\rangle_D\,, \\
\left\langle-\frac{1}{12}\left[\hat\Phi,\hat G_{-1}\hat\Phi\right]\left[\hat\Phi,\hat G_0\hat\Phi\right]\right\rangle_D &= \underbrace{-\left\langle\frac{1}{12}\left[\Phi^+,G_{-1}\Phi^+\right]\left[\Phi^+,G_0\Phi^+\right]\right\rangle_D}_{GSO(+)} - \\
&\quad - \left\langle\frac{1}{12}\left\{\Phi^-,G_{-1}\Phi^-\right\}\left\{\Phi^-,G_0\Phi^-\right\}\right\rangle_D + \\
&\quad + \left\langle\frac{1}{12}\left[\Phi^+,G_{-1}\Phi^+\right]\left\{\Phi^-,G_0\Phi^-\right\}\right\rangle_D +
\end{aligned}
\tag{5.5}
$$

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$$+\left\langle \frac{1}{12}\left\{\Phi^-, G_{-1}\Phi^-\right\}\left[\Phi^+, G_0\Phi^+\right]\right\rangle_D -$$
$$-\left\langle \frac{1}{12}\left[\Phi^+, G_{-1}\Phi^-\right]\left[\Phi^+, G_0\Phi^-\right]\right\rangle_D -$$
$$-\left\langle \frac{1}{12}\left\{\Phi^-, G_{-1}\Phi^+\right\}\left\{\Phi^-, G_0\Phi^+\right\}\right\rangle_D +$$
$$+\left\langle \frac{1}{12}\left[\Phi^+, G_{-1}\Phi^-\right]\left\{\Phi^-, G_0\Phi^+\right\}\right\rangle_D +$$
$$+\left\langle \frac{1}{12}\left\{\Phi^-, G_{-1}\Phi^+\right\}\left[\Phi^+, G_0\Phi^-\right]\right\rangle_D . \tag{5.6}$$

Note that only even number of $GSO(-)$ string fields make no vanishing contribution.

Now we need to compute these correlators. Most of the calculations are tedious but straightforward. We consider all the spin fields bosonized, which allow us to implement a routine to calculate the correlators in programs of symbolic computation to get the correct factors. Since bosonization breaks Lorentz invariance, extra care need to be taken in order to write the results in Lorentz invariant manner.

$$\begin{aligned}
\left\langle -G_{-1}\Phi^- G_0\Phi^-\right\rangle_D &= T\Box T + \frac{1}{2}T^2 + \\
&\quad +2i\left(\chi^-\sigma^m\partial_m\bar{\chi}^- + \bar{\chi}^-\bar{\sigma}^m\partial_m\chi^-\right), && (5.7)\\
\left\langle -\frac{1}{3}\hat{\Phi}\left\{\hat{G}_{-1}\hat{\Phi}, \hat{G}_0\hat{\Phi}\right\}\right\rangle_D &= 4\ 3^{3/4}T\left(\chi^+\chi^- + \bar{\chi}^+\bar{\chi}^-\right) + \\
&\quad +2\left(\chi^+\sigma^m\left[A_m, \bar{\chi}^-\right] + \bar{\chi}^-\bar{\sigma}^m\left[A_m, \chi^+\right]\right), && (5.8)\\
\left\langle -\frac{1}{12}\left\{\Phi^-, G_{-1}\Phi^-\right\}\left\{\Phi^-, G_0\Phi^-\right\}\right\rangle_D &= -T^4, && (5.9)\\
\left\langle \frac{1}{12}\left[\Phi^+, G_{-1}\Phi^+\right]\left\{\Phi^-, G_0\Phi^-\right\}\right\rangle_D &= 0, && (5.10)\\
\left\langle -\frac{1}{12}\left\{\Phi^-, G_{-1}\Phi^-\right\}\left[\Phi^+, G_0\Phi^+\right]\right\rangle_D &= 0, && (5.11)\\
\left\langle -\frac{1}{12}\left[\Phi^+, G_{-1}\Phi^-\right]\left[\Phi^+, G_0\Phi^-\right]\right\rangle_D &= \frac{1}{2}[T, A_m][T, A^m], && (5.12)\\
\left\langle \frac{1}{12}\left\{\Phi^+, G_{-1}\Phi^-\right\}\left\{\Phi^-, G_0\Phi^+\right\}\right\rangle_D &= 0, && (5.13)\\
\left\langle -\frac{1}{12}\left\{\Phi^-, G_{-1}\Phi^+\right\}\left\{\Phi^-, G_0\Phi^+\right\}\right\rangle_D &= \frac{1}{2}[T, A_m][T, A^m], && (5.14)\\
\left\langle \frac{1}{12}\left\{\Phi^-, G_{-1}\Phi^+\right\}\left[\Phi^+, G_0\Phi^-\right]\right\rangle_D &= 0. && (5.15)
\end{aligned}$$

For the second Wess-Zumino part of the action things are easier, since most of the terms vanish. Only the following term contributes

$$S^{(2)}_{WZ} = -\langle G_{-2/3}\Phi^- G_{-1/3}\Phi^-\rangle_D = 4\partial_{-i}T\partial_{+i}T\,. \tag{5.16}$$

There is still some terms in the D correlator:

$$S_D = -\left\langle e^{-\hat{\Phi}}\hat{\Omega}e^{\hat{\Phi}}\hat{\Omega}\right\rangle_D + \left\langle \hat{\Omega}e^{\hat{\Phi}}\hat{G}_{-2/3}e^{-\hat{\Phi}}\right\rangle_D + \left\langle \hat{\Omega}e^{-\hat{\Phi}}\hat{G}_{-1/3}e^{\hat{\Phi}}\right\rangle_D . \tag{5.17}$$

Splitting these terms in $GSO(+)$ and $GSO(-)$ fields as before

$$\begin{aligned}
\left\langle -e^{-\hat{\Phi}}\hat{\bar{\Omega}}e^{\hat{\Phi}}\hat{\Omega}\right\rangle_D &= \underbrace{\langle\bar{\Omega}^+\Omega^+\rangle_D}_{GSO(+)} + \langle\bar{\Omega}^-\Omega^-\rangle_D \\
&\quad -\langle\left[\Phi^+,\bar{\Omega}^-\right]\Omega^-\rangle_D - \langle\left[\Phi^-,\bar{\Omega}^-\right]\Omega^+\rangle_D + \\
&\quad +\langle\left[\Phi^-,\bar{\Omega}^+\right]\Omega^-\rangle_D - \\
&\quad -\frac{1}{2}\langle\{\Phi^-\{\Phi^-,\bar{\Omega}^+\}\}\,\Omega^+\rangle_D\,, \qquad (5.18)\\
\left\langle \hat{\bar{\Omega}}e^{\hat{\Phi}}\hat{G}_{-2/3}e^{-\hat{\Phi}}\right\rangle_D &= -\langle\bar{\Omega}^+\{\Phi^-,G_{-2/3}\Phi^-\}\rangle_D\,, \qquad (5.19)\\
\left\langle \hat{\Omega}e^{-\hat{\Phi}}\hat{G}_{-1/3}e^{\hat{\Phi}}\right\rangle_D &= \langle\Omega^+\{\Phi^-,G_{-1/3}\Phi^-\}\rangle_D\,. \qquad (5.20)
\end{aligned}$$

The correlators are calculated in the same way as before, giving us the mixing of the 4-dimensional and the internal 6-dimensional fields.

$$\begin{aligned}
\langle\bar{\Omega}^-\Omega^-\rangle_D &= -i\left(\lambda^-_{-i}\sigma^m\partial_m\bar{\lambda}^-_{-i} + \bar{\lambda}^-_{-i}\sigma^m\partial_m\lambda^-_{+i}\right), \qquad (5.21)\\
\langle-\left[\Phi^+,\bar{\Omega}^-\right]\Omega^-\rangle_D &= -2\left[A_m,\lambda^-_{+i}\right]\sigma^m\lambda^-_{+i}\,, \qquad (5.22)\\
\langle-\left[\Phi^-,\bar{\Omega}^-\right]\Omega^+\rangle_D &= 4\lambda^-_{+i}\chi^-A_{+i} - 2\,3^{3/4}T\lambda^+_{+i}\lambda^-_{-i}\,, \qquad (5.23)\\
\langle\left[\Phi^-,\bar{\Omega}^+\right]\Omega^-\rangle_D &= 4\bar{\lambda}^-_{-j}\bar{\chi}^-A_{-j} - 2\,3^{3/4}T\bar{\lambda}^-_{-i}\bar{\lambda}^+_{+i}\,, \qquad (5.24)\\
\left\langle -\frac{1}{2}\{\Phi^-\{\Phi^-,\bar{\Omega}^+\}\}\,\Omega^+\right\rangle_D &= 2T^2A_{+i}A_{-i}\,, \qquad (5.25)\\
\langle-\bar{\Omega}^+\{\Phi^-,G_{-2/3}\Phi^-\}\rangle_D &= 4\sqrt{3}A^{+i}T\partial_{+i}T\,, \qquad (5.26)\\
\langle\Omega^+\{\Phi^-,G_{-1/3}\Phi^-\}\rangle_D &= 4\sqrt{3}A^{-i}T\partial_{-i}T\,. \qquad (5.27)
\end{aligned}$$

and finally the two Chern-Simons terms contribute with:

$$\begin{aligned}
\bar{S}_{CS} &= -\left\langle\frac{1}{2}\hat{\bar{\Omega}}\hat{G}_{-1/3}\hat{\bar{\Omega}}\right\rangle_{\bar{F}} - \left\langle\frac{1}{3}\hat{\bar{\Omega}}\hat{\bar{\Omega}}\hat{\bar{\Omega}}\right\rangle_{\bar{F}}\\
&= \underbrace{-\left\langle\frac{1}{2}\bar{\Omega}^+G_{-1/3}\bar{\Omega}^+\right\rangle_{\bar{F}} - \left\langle\frac{1}{3}\bar{\Omega}^+\bar{\Omega}^+\bar{\Omega}^+\right\rangle_{\bar{F}}}_{GSO(+)} - \left\langle\frac{1}{2}\bar{\Omega}^-G_{-1/3}\bar{\Omega}^-\right\rangle_{\bar{F}} - \left\langle\frac{1}{3}\bar{\Omega}^+\bar{\Omega}^-\bar{\Omega}^-\right\rangle_{\bar{F}}\,,\\
&= -\frac{1}{2}\epsilon_{ijk}\lambda^+_{+i}\partial_{-j}\lambda^+_{+k} - \frac{1}{3}\epsilon_{ijk}A_{+i}\lambda^+_{+j}\lambda^+_{+k}\\
&\quad -\frac{1}{2}\epsilon_{ijk}\lambda^-_{+i}\partial_{-j}\lambda^-_{+k} - \frac{1}{3}\epsilon_{ijk}A_{+i}\lambda^-_{+j}\lambda^-_{+k}. \qquad (5.28)\\
S_{CS} &= \left\langle\frac{1}{2}\hat{\Omega}\hat{G}_{-2/3}\hat{\Omega}\rangle_F + \langle\frac{1}{3}\hat{\Omega}\hat{\Omega}\hat{\Omega}\right\rangle_F\\
&= \frac{1}{2}\epsilon_{ijk}\bar{\lambda}^+_{-i}\partial_{+j}\bar{\lambda}^+_{-k} + \frac{1}{3}\epsilon_{ijk}A_{-i}\bar{\lambda}^+_{-j}\bar{\lambda}^+_{-k} + \frac{1}{2}\epsilon_{ijk}\bar{\lambda}^-_{-i}\partial_{+j}\bar{\lambda}^-_{-k} + \frac{1}{3}\epsilon_{ijk}A_{-i}\bar{\lambda}^-_{-j}\bar{\lambda}^-_{-k}\,. \qquad (5.29)
\end{aligned}$$

The action we got after all this computation is simply:

$$S = \underset{\text{eq. (5.4)}}{S^{(1)}_{WZ}} + \underset{\text{eq. (5.16)}}{S^{(2)}_{WZ}} + \underset{\text{eq. (5.17)}}{S_D} + \underset{\text{eq. (5.29)}}{S_{CS}} + \underset{\text{eq. (5.28)}}{\bar{S}_{CS}}\,.$$

Now we may rewrite the 4-dimensional action obtained from all these terms using 10-dimensional fields. In order to do this we should take into account the pattern of Lorentz symmetry breaking eq. (2.12) and read, from the expressions calculated above, how they fit in explicit 10-dimensional Lorentz invariance.

The action is[2]

$$\begin{aligned} S = \mathrm{Tr} \int d^{10}x \Big[& \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + T \Box T + \frac{1}{2} T^2 + [T, A^\mu][T, A_\mu] - T^4 + \\ & + \chi^{a\,+} \gamma^\mu_{ab} \left(\partial_\mu \chi^{b\,+} + \left[A_\mu, \chi^{b\,+} \right] \right) + \chi^{a\,-} \gamma^\mu_{ab} \left(\partial_\mu \chi^{b\,-} + \left[A_\mu, \chi^{b\,-} \right] \right) + \\ & + 3^{\frac{3}{4}} T \left(\chi^{a\,+} \chi^-_a \right) + \cdots \Big] \end{aligned} \tag{5.30}$$

6. Summary and discussion

On the first part of this paper we have developed the general framework for the computation of vertices with negative GSO projection in the context of the hybrid formalism. To do that we have identified this sector with the antiperiodic sector of the hybrid variables. In general, all $GSO(-)$ vertex operator requires a hybrid spin field. This hybrid spin field turns out to be the part of zero momentum of the tachyon Λ^{++++}.

Using this framework we have explicitly identified some vertex operators for the $GSO(-)$ sector of the superstring. In particular, we have computed the vertices for tachyon and massless fermions in both sectors (Ramond sector). We did the calculation in the hybrid formalism, where the fields are split in an explicit 4-dimensional part and an internal 6-dimensional part. Again reminding that internal for our present purposed, only mean that these are fields depending on the 6 complexified coordinates.

These vertices are useful to understand supersymmetric related issues in superstring field theory, specially in what regards to SUSY breaking in tachyon condensation processes.

On the second part of this paper, after computing the action for the string field theory including up to massless terms in both GSO sectors (which amount to be a type-IIA on a non-BPS D-brane) we arrive in a full Lorentz covariant action in 10-D, including the Ramond sector. We note that our action now include the massless fermions of the action, which was not found in the previous literature.

Yoneya [2] argue that the full string field theory action including both GSO projections should have a non-linear supersymmetry since when the system undergoes Tachyon condensation we recover a closed string vacuum. This non-linear SUSY should transform fields of all masses - infinite scalars acquire vacuum expectation values in the closed string vacuum, but the existence of the tri-linear term in our action shows that we may, at least naively, consider the arguments in [2] for non-linear SUSY realizations, by considering the $GSO(-)$ fermions as goldstinos with a non-linear term in its SUSY transformation that can be calculated order by order. Of course this approach is hopeless in the current presentation of the problem since the full action involves an infinite number of terms, however, this questions are still under work.

[2]See footnote in the introduction

	b	c	ξ	η	$e^{q\phi}$	x^m	ψ^m	ψ^{+j}	ψ^{-j}	e^{qH}	x^{+j}	x^{-j}
C	0	0	1	-1	q	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	q	0	0
P	0	0	1	-1	q	0	0	0	0	0	0	0
J_{gh}	-1	1	-1	1	0	0	0	0	0	0	0	0
U(1)	0	0	0	0	0	0	0	1	-1	q	0	0
W	2	-1	0	1	$-\frac{1}{2}q(q+2)$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}q^2$	0	0
Statistic	F	F	F	F		B	F	F	F		B	B

Table 2: Properties of the Extended RNS Variables.

One may try to consider some other possibilities, including the vacuum superstring field theory in the presence of fermions or Boundary String Field Theory. After the present first step, these issues are under investigation. Any other approach used may rely on this calculation by considering the suitable field redefinitions.

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A. RNS variables

$N=1,\ c=15$ superconformal generators $T_m = -\frac{1}{2}\partial x^\mu \partial x_\mu - \frac{1}{2}\psi^\mu \partial \psi_\mu,\ \ G_m = i\psi^\mu \partial x_\mu$.

BRST operator

$$Q = \int \left[c(T_m - b\partial c - \partial^2\phi - \frac{1}{2}(\partial\phi)^2 - \eta\partial\xi) + \eta e^{\phi} G_m - \eta\partial\eta e^{2\phi} b \right].$$

- Picture Raising: $Z = e^{\phi}G_m + b\partial\eta e^{2\phi} + \partial(b\eta e^{2\phi}) + c\partial\xi$.
- Picture Lowering: $Y = c\partial\xi e^{-2\phi}$.
- C-Charge: $C = P + \frac{i}{3}\int dz(\psi^4\psi^5 + \psi^6\psi^7 + \psi^8\psi^9)$.
- Picture Number: $P = \int dz(\xi\eta - \partial\phi)$.
- Ghost Charge: $J_{gh} = \int dz(\eta\xi + cb)$.

Non-vanishing inner product

$$\langle \xi e^{-2\phi} c\partial c\partial^2 c\rangle = 1\,.$$

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Hermiticity Relation $j_{BRST} = b^\dagger$

$$\begin{aligned} j_{BRST} &= e^{-K}(-b\eta\partial\eta e^{2\phi})e^{K}\,, \\ K &= \int\left[c\xi e^{-\phi}\psi^\mu\partial x_\mu + \frac{1}{2}\partial\phi c\partial c\xi\partial\xi e^{-2\phi}\right]. \end{aligned}$$

Bosonization of ψ^m

$$\begin{aligned} \psi_1^\pm \equiv e^{\pm\tau_1} &\cong \frac{1}{\sqrt{2}}\left(\psi^3 \pm \psi^0\right), \\ \psi_2^\pm \equiv e^{\pm\tau_2} &\cong \frac{1}{\sqrt{2}}\left(\psi^1 \pm i\psi^2\right). \end{aligned}$$

Bosonization of $\psi^{\pm j}$

$$\psi^{\pm j} \equiv e^{\pm\tau_{j+2}} \cong \frac{1}{\sqrt{2}}\left(\psi^{2j+2} \pm i\psi^{2j+3}\right); \qquad j = 1,2,3\,.$$

Four dimensional RNS spin fields Σ^α and $\bar\Sigma^{\dot\alpha}$

$$\begin{aligned} \Sigma^\alpha \equiv \Sigma^{\pm\pm} &\equiv e^{\pm\frac{1}{2}\tau_1\pm\frac{1}{2}\tau_2}\,, \qquad (even\ +)\,, \\ \bar\Sigma^{\dot\alpha} \equiv \Sigma^{\pm\pm} &\equiv e^{\pm\frac{1}{2}\tau_1\pm\frac{1}{2}\tau_2}\,, \qquad (odd\ +)\,. \end{aligned} \tag{A.1}$$

Four-dimensional spinor index

α	$\dot\alpha$
$1 \equiv ++$	$\dot 1 \equiv +-$
$2 \equiv --$	$\dot 2 \equiv -+$

Internal RNS Spin Fields: Ξ^a and $\bar\Xi^{\dot a}$

$$\begin{aligned} \Xi^a \equiv \Xi^{\pm\pm\pm} &\equiv e^{\pm\frac{1}{2}\tau_3\pm\frac{1}{2}\tau_4\pm\frac{1}{2}\tau_5}\,, \qquad (even\ +)\,, \\ \bar\Xi^{\dot a} \equiv \Xi^{\pm\pm\pm} &\equiv e^{\pm\frac{1}{2}\tau_3\pm\frac{1}{2}\tau_4\pm\frac{1}{2}\tau_5}\,, \qquad (odd\ +)\,. \end{aligned} \tag{A.2}$$

Internal spinor indices a (chiral), $\dot a$ (anti-chiral)

Ξ^a		$\Xi^{\dot a}$	
$e^{H/2}$	Ξ^{---}	$e^{-H/2}$	Ξ^{+++}
Ξ^{+j}	Ξ^{-++}	Ξ^{-j}	Ξ^{+--}
	Ξ^{+-+}		Ξ^{-+-}
	Ξ^{++-}		Ξ^{--+}

B. Hybrid variables

C-Charge: $C = \frac{1}{3}\int dz\Gamma^{+j}\Gamma^{-j}$.

Ghost Charge $J_{gh} = \int dz(\partial\rho + \Gamma^{-j}\Gamma^{+j})$.

Non-vanishing inner product

$$\left\langle \frac{1}{24}(\theta)^2(\bar\theta)^2 e^{-\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}\right\rangle = 1\,.$$

	X^m	p	θ	$\bar{p}$	$\bar{\theta}$	$e^{q\rho}$	Γ^{+j}	Γ^{-j}	X^{+j}	X^{+j}
C	0	0	0	0	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	0
P	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	q	0	0	0	0
J_{gh}	0	0	0	0	0	q	-1	1	0	0
ρ	0	0	0	0	0	q	0	0	0	0
W	0	1	0	1	0	$-\frac{1}{2}q(q+1)$	1	0	0	0
Statistic	B	F	F	F	F		F	F	B	B

Table 3: Properties of the Hybrid Variables.

Normal ordering

$$\begin{aligned} d^\alpha d_\alpha(z) = &\, p^\alpha p_\alpha(z) + p^\alpha \sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} i\partial X_m(z) + \frac{1}{8}\partial(\bar\theta)^2\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}\theta_\alpha i\partial X_m(z) + \\ &+\frac{1}{64}(\theta)^2\partial(\bar\theta)^2\partial(\bar\theta)^2(z) - \frac{1}{2}(\bar\theta)^2 : p^\alpha\partial\theta_\alpha(z) : +\frac{1}{4}\partial(\bar\theta)^2 : p^\alpha\theta_\alpha(z) : + \\ &+\frac{1}{4}(\bar\theta)^2 : \partial X^m\partial X_m(z) : -\frac{1}{2}\partial\bar\theta_{\dot\alpha}\partial\bar\theta^{\dot\alpha}(z) + \frac{1}{2}\partial^2\bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}(z)\,. \end{aligned} \tag{B.1}$$

The Hermiticity property of the hybrid variables

$$\begin{aligned} (X^m)^\dagger &= X^m\,, \\ \theta_\alpha^\dagger &= \bar\theta_{\dot\alpha}\,, \\ p_\alpha^\dagger &= -\bar p_{\dot\alpha}\,, \\ (\Gamma^{-j})^\dagger &= \frac{1}{2}\epsilon^{jkl}\Gamma^{-k}\Gamma^{-l}e^{-\rho}\,, \\ (\Gamma^{+j})^\dagger &= \frac{1}{2}\epsilon^{jkl}\Gamma^{+k}\Gamma^{+l}e^{\rho}\,, \\ (e^\rho)^\dagger &= \frac{1}{6}e^{-2\rho}\epsilon^{jkl}\Gamma^{-j}\Gamma^{-k}\Gamma^{-l}\,, \\ (X^{-j})^\dagger &= X^{-j}\,, \\ (X^{+j})^\dagger &= X^{+j}\,. \end{aligned} \tag{B.2}$$

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